

Super w_∞ 3-algebra

Min-Ru Chen^{a,b}, Ke Wu^{a,1} and Wei-Zhong Zhao^{a,c,2}

^a*School of Mathematical Sciences, Capital Normal University, Beijing 100048, China*

^b*College of Mathematics and Information Sciences, Henan University, Kaifeng 475001, China*

^c*Institute of Mathematics and Interdisciplinary Science, Capital Normal University, Beijing 100048, China*

Abstract

We investigate the super high-order Virasoro 3-algebra. By applying the appropriate scaling limits on the generators, we obtain the super w_∞ 3-algebra which satisfies the generalized fundamental identity condition. We also define a super Nambu-Poisson bracket which satisfies the generalized skewsymmetry, Leibniz rule and fundamental identity. By means of this super Nambu-Poisson bracket, the realization of the super w_∞ 3-algebra is presented.

KEYWORDS: 3-algebra, W symmetry, Supersymmetry.

PACS numbers: 02.10.De, 02.20.Sv, 11.25.Hf, 11.25.-w

¹ wuke@mail.cnu.edu.cn

² zhaowz100@163.com

1 Introduction

Since Nambu [1] first proposed 3-bracket for the generalized Hamiltonian dynamics, 3-algebras have attracted much interest from physical and mathematical points of view. Recently with the development of string theory, it is found that 3-algebras have the important applications in M-theory, such as multiple M2-branes [2]-[6] and Nambu-Poisson M5-brane theory [7]-[9]. 3-algebras can be realized in two ways, i.e., classical Nambu and quantal brackets. These two brackets are defined by means of multi-variable Jacobians and antisymmetrized products of three linear operators, respectively. The properties of various 3-algebras have been widely investigated [9]-[12]. The Virasoro algebra is an infinite-dimensional algebra which plays an important role in conformal field theory. The centerless Virasoro 3-algebra, i.e., Virasoro-Witt 3-algebra has been constructed in the literature [13]-[15]. The W_∞ algebra is the higher-spin extensions of the Virasoro algebra [16]. Quite recently by applying a double scaling limits on the generators of the W_∞ algebra, Chakraborty et al.[17] constructed a w_∞ 3-algebra which satisfies the fundamental identity (FI) condition of 3-algebra.

The supersymmetric generalizations of 3-algebras are of general interest [18]-[23]. Recently Sakakibara [21] constructed the super Nambu-Poisson algebra and demonstrated its connection with the generalized Batalin-Vilkovisky algebra. Soroka et al. proposed the Grassmann-odd Nambu brackets and investigated their properties in their serial paper [22][23]. The supersymmetric generalizations of the Virasoro and w algebras have been well investigated. As to their super 3-algebras, to our best knowledge, it has not been reported so far in the existing literature. The propose of this paper is to present the super w_∞ 3-algebra which satisfies the generalized FI condition.

This paper is organized as follows. In the next section, we investigate the super high-order Virasoro (SHOV) algebra and its 3-algebra. Then by applying an appropriate scaling limits on the generators of the SHOV algebra, we give the super w_∞ 3-algebra. In section 3, we define the super Nambu-Poisson bracket to realize the super w_∞ 3-algebra. We end this paper with the concluding remarks in section 4.

2 SHOV algebra and super w_∞ 3-algebra

The generators of SHOV algebra are given by

$$\begin{aligned} L_m^i &= (-1)^i \lambda^{i-\frac{1}{2}} z^{m+i} \partial_z^i, \\ \bar{L}_m^i &= (-1)^i \lambda^{i+\frac{3}{2}} z^{m+i} \theta \partial_\theta \partial_z^i, \\ h_r^{\alpha+\frac{1}{2}} &= (-1)^{\alpha+1} \lambda^{\alpha+\frac{1}{2}} z^{r+\alpha} \partial_\theta \partial_z^\alpha, \\ \bar{h}_r^{\alpha+\frac{1}{2}} &= (-1)^{\alpha+1} \lambda^{\alpha+\frac{1}{2}} z^{r+\alpha} \theta \partial_z^\alpha, \end{aligned} \tag{1}$$

where $i, \alpha \in Z_+$, $m, r \in Z$ and λ is an arbitrary parameter.

Their communication relations are

$$\begin{aligned} [L_m^i, L_n^j] &= \sum_{p=0}^i (-1)^p \lambda^{p-\frac{1}{2}} C_i^p B_p^{n+j} L_{m+n}^{i+j-p} - \sum_{p=0}^j (-1)^p \lambda^{p-\frac{1}{2}} C_j^p B_p^{m+i} L_{m+n}^{i+j-p}, \\ [L_m^i, \bar{L}_n^j] &= \sum_{p=0}^i (-1)^p \lambda^{p-\frac{1}{2}} C_i^p B_p^{n+j} \bar{L}_{m+n}^{i+j-p} - \sum_{p=0}^j (-1)^p \lambda^{p-\frac{1}{2}} C_j^p B_p^{m+i} \bar{L}_{m+n}^{i+j-p}, \end{aligned}$$

$$[L_m^i, h_r^{\alpha+\frac{1}{2}}] = \sum_{p=0}^i (-1)^p \lambda^{p-\frac{1}{2}} C_i^p B_p^{r+\alpha} h_{m+r}^{\alpha+i-p+\frac{1}{2}} - \sum_{p=0}^{\alpha} (-1)^p \lambda^{p-\frac{1}{2}} C_{\alpha}^p B_p^{m+i} h_{m+r}^{\alpha+i-p+\frac{1}{2}},$$

$$[L_m^i, \bar{h}_r^{\alpha+\frac{1}{2}}] = \sum_{p=0}^i (-1)^p \lambda^{p-\frac{1}{2}} C_i^p B_p^{r+\alpha} \bar{h}_{m+r}^{\alpha+i-p+\frac{1}{2}} - \sum_{p=0}^{\alpha} (-1)^p \lambda^{p-\frac{1}{2}} C_{\alpha}^p B_p^{m+i} \bar{h}_{m+r}^{\alpha+i-p+\frac{1}{2}},$$

$$[\bar{L}_m^i, h_r^{\alpha+\frac{1}{2}}] = - \sum_{p=0}^{\alpha} (-1)^p \lambda^{p+\frac{3}{2}} C_{\alpha}^p B_p^{m+i} h_{m+r}^{\alpha+i-p+\frac{1}{2}},$$

$$[\bar{L}_m^i, \bar{L}_n^j] = \sum_{p=0}^i (-1)^p \lambda^{p+\frac{3}{2}} C_i^p B_p^{n+j} \bar{L}_{m+n}^{i+j-p} - \sum_{p=0}^j (-1)^p \lambda^{p+\frac{3}{2}} C_j^p B_p^{m+i} \bar{L}_{m+n}^{i+j-p},$$

$$[\bar{L}_m^i, \bar{h}_r^{\alpha+\frac{1}{2}}] = \sum_{p=0}^i (-1)^p \lambda^{p+\frac{3}{2}} C_i^p B_p^{r+\alpha} \bar{h}_{m+r}^{\alpha+i-p+\frac{1}{2}},$$

$$\begin{aligned} [h_r^{\alpha+\frac{1}{2}}, \bar{h}_s^{\beta+\frac{1}{2}}] &= \sum_{p=0}^{\alpha} (-1)^p \lambda^{p+\frac{3}{2}} C_{\alpha}^p B_p^{s+\beta} L_{r+s}^{\alpha+\beta-p} + \sum_{p=0}^{\beta} (-1)^p \lambda^{p-\frac{1}{2}} C_{\beta}^p B_p^{r+\alpha} \bar{L}_{r+s}^{\alpha+\beta-p} \\ &- \sum_{p=0}^{\alpha} (-1)^p \lambda^{p-\frac{1}{2}} C_{\alpha}^p B_p^{s+\beta} \bar{L}_{r+s}^{\alpha+\beta-p}, \end{aligned} \quad (2)$$

where $B_p^n = \begin{cases} n(n-1) \cdots (n-p+1), & p \leq n \\ 0, & p > n, \end{cases}$ $C_n^m = \frac{n(n-1) \cdots (n-m+1)}{m!}$ and the communication relation is defined by

$$[f, g] = fg - (-1)^{|f||g|} gf, \quad (3)$$

$|f|$ and $|g|$ are the parity of f and g , respectively. A notational convention used frequently in the rest of this paper is that for any arbitrary h , the symbol $|h|$ appearing in the exponent of (-1) is to be understood as the parity of h . When $\lambda = 1$, (2) leads to the communication relations of SHOV algebra derived by Zha and Zhao [24]. Recently Chakraborty et al. [17] constructed a w_{∞} 3-algebra by applying a double scaling limits on the generators of W_{∞} algebra. In order to construct the super w_{∞} 3-algebra, we introduce a parameter λ into the generators of SHOV algebra (1). Not as done by Chakraborty et al., we'll construct the super w_{∞} 3-algebra by taking a single scaling limit on the generators (1). Therefore the parameter λ plays a crucial role in the following investigation.

Let us define a super 3-bracket as follows:

$$[f, g, h] = [f, g]h + (-1)^{|f|(|g|+|h|)} [g, h]f + (-1)^{|h|(|f|+|g|)} [h, f]g, \quad (4)$$

where the commutator $[,]$ is defined by (3). Substituting the generators (1) into the generalized ternary commutator (4), we may obtain the SHOV 3-algebra. Due to too many 3-algebra

relations, we only list some of them that will be used in the late discussion.

$$\begin{aligned}
[L_m^i, L_n^j, L_k^h] &= [(\sum_{p=0}^i C_i^p B_p^{n+j} - \sum_{p=0}^j C_j^p B_p^{m+i}) \sum_{q=0}^{i+j-p} C_{i+j-p}^q B_q^{k+h} + (\sum_{p=0}^j C_j^p B_p^{k+h} \\
&\quad - \sum_{p=0}^h C_h^p B_p^{n+j}) \sum_{q=0}^{j+h-p} C_{j+h-p}^q B_q^{m+i} + (\sum_{p=0}^h C_h^p B_p^{m+i} \\
&\quad - \sum_{p=0}^i C_i^p B_p^{k+h}) \sum_{q=0}^{h+i-p} C_{h+i-p}^q B_q^{n+j}] (-1)^{p+q} \lambda^{p+q-1} L_{m+n+k}^{i+j+h-p-q},
\end{aligned}$$

$$\begin{aligned}
[L_m^i, L_n^j, \bar{L}_k^h] &= [(\sum_{p=0}^i C_i^p B_p^{n+j} - \sum_{p=0}^j C_j^p B_p^{m+i}) \sum_{q=0}^{i+j-p} C_{i+j-p}^q B_q^{k+h} + (\sum_{p=0}^j C_j^p B_p^{k+h} \\
&\quad - \sum_{p=0}^h C_h^p B_p^{n+j}) \sum_{q=0}^{j+h-p} C_{j+h-p}^q B_q^{m+i} + (\sum_{p=0}^h C_h^p B_p^{m+i} \\
&\quad - \sum_{p=0}^i C_i^p B_p^{k+h}) \sum_{q=0}^{h+i-p} C_{h+i-p}^q B_q^{n+j}] (-1)^{p+q} \lambda^{p+q-1} \bar{L}_{m+n+k}^{i+j+h-p-q},
\end{aligned}$$

$$\begin{aligned}
[L_m^i, L_n^j, h_r^{\alpha+\frac{1}{2}}] &= [(\sum_{p=0}^i C_i^p B_p^{n+j} - \sum_{p=0}^j C_j^p B_p^{m+i}) \sum_{q=0}^{i+j-p} C_{i+j-p}^q B_q^{r+\alpha} + (\sum_{p=0}^j C_j^p B_p^{r+\alpha} \\
&\quad - \sum_{p=0}^{\alpha} C_{\alpha}^p B_p^{n+j}) \sum_{q=0}^{j+\alpha-p} C_{j+\alpha-p}^q B_q^{m+i} + (\sum_{p=0}^{\alpha} C_{\alpha}^p B_p^{m+i} \\
&\quad - \sum_{p=0}^i C_i^p B_p^{r+\alpha}) \sum_{q=0}^{\alpha+i-p} C_{\alpha+i-p}^q B_q^{n+j}] (-1)^{p+q} \lambda^{p+q-1} h_{m+n+r}^{i+j+\alpha-p-q+\frac{1}{2}},
\end{aligned}$$

$$\begin{aligned}
[L_m^i, L_n^j, \bar{h}_r^{\alpha+\frac{1}{2}}] &= [(\sum_{p=0}^i C_i^p B_p^{n+j} - \sum_{p=0}^j C_j^p B_p^{m+i}) \sum_{q=0}^{i+j-p} C_{i+j-p}^q B_q^{r+\alpha} + (\sum_{p=0}^j C_j^p B_p^{r+\alpha} \\
&\quad - \sum_{p=0}^{\alpha} C_{\alpha}^p B_p^{n+j}) \sum_{q=0}^{j+\alpha-p} C_{j+\alpha-p}^q B_q^{m+i} + (\sum_{p=0}^{\alpha} C_{\alpha}^p B_p^{m+i} \\
&\quad - \sum_{p=0}^i C_i^p B_p^{r+\alpha}) \sum_{q=0}^{\alpha+i-p} C_{\alpha+i-p}^q B_q^{n+j}] (-1)^{p+q} \lambda^{p+q-1} \bar{h}_{m+n+r}^{i+j+\alpha-p-q+\frac{1}{2}},
\end{aligned}$$

$$\begin{aligned}
[L_m^i, h_r^{\alpha+\frac{1}{2}}, \bar{h}_s^{\beta+\frac{1}{2}}] &= [(\sum_{p=0}^i C_i^p B_p^{r+\alpha} - \sum_{p=0}^\alpha C_\alpha^p B_p^{m+i}) \sum_{q=0}^{i+\alpha-p} C_{i+\alpha-p}^q B_q^{s+\beta} \\
&+ \sum_{p=0}^\alpha \sum_{q=0}^{\alpha+\beta-p} C_\alpha^p B_p^{s+\beta} C_{\alpha+\beta-p}^q B_q^{m+i}] (-1)^{p+q} \lambda^{p+q+1} L_{m+r+s}^{i+\alpha+\beta-p-q} \\
&+ [-(\sum_{p=0}^i C_i^p B_p^{r+\alpha} - \sum_{p=0}^\alpha C_\alpha^p B_p^{m+i}) \sum_{q=0}^{i+\alpha-p} C_{i+\alpha-p}^q B_q^{s+\beta} + (\sum_{p=0}^\beta C_\beta^p B_p^{r+\alpha} \\
&- \sum_{p=0}^\alpha C_\alpha^p B_p^{s+\beta}) \sum_{q=0}^{\alpha+\beta-p} C_{\alpha+\beta-p}^q B_q^{m+i} - (\sum_{p=0}^\beta C_\beta^p B_p^{m+i} \\
&- \sum_{p=0}^i C_i^p B_p^{s+\beta}) \sum_{q=0}^{i+\beta-p} C_{i+\beta-p}^q B_q^{r+\alpha}] (-1)^{p+q} \lambda^{p+q-1} \bar{L}_{m+r+s}^{i+\alpha+\beta-p-q}. \quad (5)
\end{aligned}$$

Let us take the scaling limit $\lambda \rightarrow 0$ and for convenience denote the generators with the same notations for this and other kinds of limit throughout this paper, then from (5), we obtain the following super w_∞ 3-algebra:

$$\begin{aligned}
[L_m^i, L_n^j, L_k^h] &= (h(n-m) + j(m-k) + i(k-n)) L_{m+n+k}^{i+j+h-1}, \\
[L_m^i, L_n^j, \bar{L}_k^h] &= -[L_m^i, \bar{L}_k^h, L_n^j] = [\bar{L}_k^h, L_m^i, L_n^j] \\
&= (h(n-m) + j(m-k) + i(k-n)) \bar{L}_{m+n+k}^{i+j+h-1}, \\
[L_m^i, L_n^j, h_r^{\alpha+\frac{1}{2}}] &= -[L_m^i, h_r^{\alpha+\frac{1}{2}}, L_n^j] = [h_r^{\alpha+\frac{1}{2}}, L_m^i, L_n^j] \\
&= (\alpha(n-m) + j(m-r) + i(r-n)) h_{m+n+r}^{i+j+\alpha-1+\frac{1}{2}}, \\
[L_m^i, L_n^j, \bar{h}_r^{\alpha+\frac{1}{2}}] &= -[L_m^i, \bar{h}_r^{\alpha+\frac{1}{2}}, L_n^j] = [\bar{h}_r^{\alpha+\frac{1}{2}}, L_m^i, L_n^j] \\
&= (\alpha(n-m) + j(m-r) + i(r-n)) \bar{h}_{m+n+r}^{i+j+\alpha-1+\frac{1}{2}}, \\
[L_m^i, h_r^{\alpha+\frac{1}{2}}, \bar{h}_s^{\beta+\frac{1}{2}}] &= [L_m^i, \bar{h}_s^{\beta+\frac{1}{2}}, h_r^{\alpha+\frac{1}{2}}] = [h_r^{\alpha+\frac{1}{2}}, \bar{h}_s^{\beta+\frac{1}{2}}, L_m^i] = [\bar{h}_s^{\beta+\frac{1}{2}}, h_r^{\alpha+\frac{1}{2}}, L_m^i] \\
&= -[h_r^{\alpha+\frac{1}{2}}, L_m^i, \bar{h}_s^{\beta+\frac{1}{2}}] = -[\bar{h}_s^{\beta+\frac{1}{2}}, L_m^i, h_r^{\alpha+\frac{1}{2}}] \\
&= (i(r-s) + \alpha(s-m) + \beta(m-r)) \bar{L}_{m+r+s}^{i+\alpha+\beta-1}, \quad (6)
\end{aligned}$$

with all other 3-brackets vanishing. It should be pointed out that under this kind of scaling limit we have the non-null 3-algebra (6), but (2) becomes the null algebra. Note that the first 3-algebraic relation in (6) is the so-called w_∞ 3-algebra derived in Ref.[17]. It is known that this w_∞ 3-algebra satisfies the usually FI condition. As to the super w_∞ 3-algebra (6), it should satisfy the generalized FI condition due to the involution of fermionic generators. We find that the super w_∞ 3-algebra (6) satisfies the following generalized FI condition:

$$\begin{aligned}
[A, B, [C, D, E]] &= [[A, B, C], D, E] + (-1)^{(|A|+|B|)|C|} [C, [A, B, D], E] \\
&+ (-1)^{(|A|+|B|)(|C|+|D|)} [C, D, [A, B, E]]. \quad (7)
\end{aligned}$$

When the generators in (7) are bosonic, (7) reduces to the usually FI condition of 3-algebra.

3 Super Nambu-Poisson bracket

For the graded communicating bracket (3), it is well-known that the classical limit is given by

$$\{ , \} = \lim_{\hbar \rightarrow 0} \frac{1}{\mathbf{i}\hbar} [,]. \quad (8)$$

Let us take the generators of SHOV algebra as follows:

$$\begin{aligned} L_m^i &= (-\mathbf{i}\hbar)^i z^{m+i} \partial_z^i, & \bar{L}_m^i &= (-\mathbf{i}\hbar)^{i+2} z^{m+i} \theta \partial_\theta \partial_z^i, \\ h_r^{\alpha+\frac{1}{2}} &= (-\mathbf{i}\hbar)^{\alpha+1} z^{r+\alpha} \partial_\theta \partial_z^\alpha, & \bar{h}_r^{\alpha+\frac{1}{2}} &= (-\mathbf{i}\hbar)^{\alpha+1} z^{r+\alpha} \theta \partial_z^\alpha, \end{aligned} \quad (9)$$

where $\mathbf{i} = \sqrt{-1}$ and \hbar is introduced in the above generators. Substituting the generators (9) into (3) and taking the limit (8), we obtain the classical super w_∞ algebra

$$\begin{aligned} \{L_m^i, L_n^j\} &= (mj - ni) L_{m+n}^{i+j-1}, \\ \{L_m^i, \bar{L}_n^j\} &= (mj - ni) \bar{L}_{m+n}^{i+j-1}, \\ \{L_m^i, h_r^{\alpha+\frac{1}{2}}\} &= (m\alpha - ri) h_{m+r}^{i+\alpha-\frac{1}{2}}, \\ \{L_m^i, \bar{h}_r^{\alpha+\frac{1}{2}}\} &= (m\alpha - ri) \bar{h}_{m+r}^{i+\alpha-\frac{1}{2}}, \\ \{\bar{L}_m^i, h_r^{\alpha+\frac{1}{2}}\} &= \{\bar{L}_m^i, \bar{h}_r^{\alpha+\frac{1}{2}}\} = \{\bar{L}_m^i, \bar{L}_n^j\} = 0, \\ \{h_r^{\alpha+\frac{1}{2}}, \bar{h}_s^{\beta+\frac{1}{2}}\} &= (s\alpha - r\beta) \bar{L}_{r+s}^{\alpha+\beta-1}. \end{aligned} \quad (10)$$

This super w_∞ algebra is different with ones derived by Pope and Shen [25].

Let us turn to discuss the case of 3-bracket. For the 3-bracket defined by (4), we require the following classical limit to be hold [10]:

$$\{ , , \} = \lim_{\hbar \rightarrow 0} \frac{1}{\mathbf{i}\hbar} [, ,], \quad (11)$$

where $\{ , , \}$ should be understood as the super Nambu-Poisson bracket.

Substituting the generators (9) into (4) and taking the classical limit (11), we obtain the classical super w_∞ 3-algebra

$$\begin{aligned} \{L_m^i, L_n^j, L_k^h\} &= (h(n-m) + j(m-k) + i(k-n)) L_{m+n+k}^{i+j+h-1}, \\ \{L_m^i, L_n^j, \bar{L}_k^h\} &= -\{L_m^i, \bar{L}_k^h, L_n^j\} = \{\bar{L}_k^h, L_m^i, L_n^j\} \\ &= (h(n-m) + j(m-k) + i(k-n)) \bar{L}_{m+n+k}^{i+j+h-1}, \\ \{L_m^i, L_n^j, h_r^{\alpha+\frac{1}{2}}\} &= -\{L_m^i, h_r^{\alpha+\frac{1}{2}}, L_n^j\} = \{h_r^{\alpha+\frac{1}{2}}, L_m^i, L_n^j\} \\ &= (\alpha(n-m) + j(m-r) + i(r-n)) h_{m+n+r}^{i+j+\alpha-1+\frac{1}{2}}, \\ \{L_m^i, L_n^j, \bar{h}_r^{\alpha+\frac{1}{2}}\} &= -\{L_m^i, \bar{h}_r^{\alpha+\frac{1}{2}}, L_n^j\} = \{\bar{h}_r^{\alpha+\frac{1}{2}}, L_m^i, L_n^j\} \\ &= (\alpha(n-m) + j(m-r) + i(r-n)) \bar{h}_{m+n+r}^{i+j+\alpha-1+\frac{1}{2}}, \end{aligned}$$

$$\begin{aligned}
\{L_m^i, h_r^{\alpha+\frac{1}{2}}, \bar{h}_s^{\beta+\frac{1}{2}}\} &= \{L_m^i, \bar{h}_s^{\beta+\frac{1}{2}}, h_r^{\alpha+\frac{1}{2}}\} = \{h_r^{\alpha+\frac{1}{2}}, \bar{h}_s^{\beta+\frac{1}{2}}, L_m^i\} = \{\bar{h}_s^{\beta+\frac{1}{2}}, h_r^{\alpha+\frac{1}{2}}, L_m^i\} \\
&= -\{h_r^{\alpha+\frac{1}{2}}, L_m^i, \bar{h}_s^{\beta+\frac{1}{2}}\} = -\{\bar{h}_s^{\beta+\frac{1}{2}}, L_m^i, h_r^{\alpha+\frac{1}{2}}\} \\
&= (i(r-s) + \alpha(s-m) + \beta(m-r)) \bar{L}_{m+r+s}^{i+\alpha+\beta-1},
\end{aligned} \tag{12}$$

with all other 3-brackets vanishing, which exactly match with (6).

The concept of quantization plays a fundamental role in quantum physics. It is well-known that the limit relation (8) establishes the relation between classical and quantum mechanics. The limit relation (11) can be regarded as the analog of (8). Takhtajan [10] investigated the quantization of Nambu mechanics based on (11). In the previous section, we pointed out that by applying the scaling limit $\lambda \rightarrow 0$ on the generators of SHOV algebra (1), we have the null algebra and non-null super w_∞ 3-algebra. When we introduce \hbar in the generators of SHOV algebra, under the limits (8) and (11), we find that the non-null super w_∞ algebra and 3-algebra can be obtained, respectively.

To present the realization of the classical super w_∞ 3-algebra (12), we define the following super Nambu-Poisson bracket:

$$\begin{aligned}
\{f, g, h\} &= \frac{\partial f}{\partial x_1} \frac{\partial g}{\partial x_2} \frac{\partial h}{\partial x_3} - \frac{\partial f}{\partial x_1} \frac{\partial g}{\partial x_3} \frac{\partial h}{\partial x_2} + \frac{\partial f}{\partial x_2} \frac{\partial g}{\partial x_3} \frac{\partial h}{\partial x_1} \\
&\quad - \frac{\partial f}{\partial x_2} \frac{\partial g}{\partial x_1} \frac{\partial h}{\partial x_3} + \frac{\partial f}{\partial x_3} \frac{\partial g}{\partial x_1} \frac{\partial h}{\partial x_2} - \frac{\partial f}{\partial x_3} \frac{\partial g}{\partial x_2} \frac{\partial h}{\partial x_1} \\
&= \sum_{\sigma} (-1)^{\varepsilon(\sigma)} \frac{\partial f}{\partial x_{\sigma(1)}} \frac{\partial g}{\partial x_{\sigma(2)}} \frac{\partial h}{\partial x_{\sigma(3)}},
\end{aligned} \tag{13}$$

where f, g and h are the functions of the bosonic variables $x_i, i = 1, 2, 3$ and Grassmann variables θ_1 and θ_2 , $\sigma \in \text{Symm}(3)$, $\text{Symm}(3)$ is a symmetric group of 3 elements and $\varepsilon(\sigma)$ is the parity of permutation σ . It should be pointed out that the derivative term with respect to the Grassmann variables does not be introduced in (13).

It is known that the bosonic Nambu bracket is characterized by the three properties, i.e., skewsymmetry, the Leibniz rule and the fundamental identity. For the super Nambu-Poisson bracket (13), these three properties should be generalized. According to (13), it is easy to prove that the following generalized skew-symmetric properties are satisfied:

$$\begin{aligned}
\{g, f, h\} &= (-1)^{1+|f||g|} \{f, g, h\}, \\
\{f, h, g\} &= (-1)^{1+|g||h|} \{f, g, h\}, \\
\{h, g, f\} &= (-1)^{1+|f||g|+|f||h|+|g||h|} \{f, g, h\}.
\end{aligned} \tag{14}$$

Let us consider the 3-bracket $\{f_1 f_2, g, h\}$. By means of (13), we obtain

$$\begin{aligned}
\{f_1 f_2, g, h\} &= \sum_{\sigma} (-1)^{\varepsilon(\sigma)} \left(\frac{\partial f_1}{\partial x_{\sigma(1)}} f_2 + f_1 \frac{\partial f_2}{\partial x_{\sigma(1)}} \right) \frac{\partial g}{\partial x_{\sigma(2)}} \frac{\partial h}{\partial x_{\sigma(3)}} \\
&= \sum_{\sigma} (-1)^{\varepsilon(\sigma)} (-1)^{|f_1||f_2|} f_2 \frac{\partial f_1}{\partial x_{\sigma(1)}} \frac{\partial g}{\partial x_{\sigma(2)}} \frac{\partial h}{\partial x_{\sigma(3)}} \\
&\quad + \sum_{\sigma} (-1)^{\varepsilon(\sigma)} f_1 \frac{\partial f_2}{\partial x_{\sigma(1)}} \frac{\partial g}{\partial x_{\sigma(2)}} \frac{\partial h}{\partial x_{\sigma(3)}} \\
&= f_1 \{f_2, g, h\} + (-1)^{|f_1||f_2|} f_2 \{f_1, g, h\}.
\end{aligned} \tag{15}$$

In a similar way, we have another two Leibniz rule relations

$$\begin{aligned}
\{f, g_1 g_2, h\} &= (-1)^{|g_2|(|f|+|g_1|)} g_2 \{f, g_1, h\} + (-1)^{|g_1||f|} g_1 \{f, g_2, h\} \\
&= (-1)^{|g_2||h|} \{f, g_1, h\} g_2 + (-1)^{|g_1||f|} g_1 \{f, g_2, h\},
\end{aligned} \tag{16}$$

$$\begin{aligned}
\{f, g, h_1 h_2\} &= \{f, g, h_1\} h_2 + (-1)^{|h_1|(|f|+|g|)} h_1 \{f, g, h_2\} \\
&= \{f, g, h_1\} h_2 + (-1)^{|h_1||h_2|} \{f, g, h_2\} h_1.
\end{aligned} \tag{17}$$

To discuss the fundamental identity, we first give the following relations by means of (13):

$$\begin{aligned}
\{A, B, \{C, D, E\}\} &= \{A, B, \sum_{\sigma'} (-1)^{\varepsilon(\sigma')} \frac{\partial C}{\partial x_{\sigma'(1)}} \frac{\partial D}{\partial x_{\sigma'(2)}} \frac{\partial E}{\partial x_{\sigma'(3)}}\} \\
&= \sum_{\sigma, \sigma'} (-1)^{\varepsilon(\sigma) + \varepsilon(\sigma')} \frac{\partial A}{\partial x_{\sigma(1)}} \frac{\partial B}{\partial x_{\sigma(2)}} \left(\frac{\partial^2 C}{\partial x_{\sigma'(1)} \partial x_{\sigma(3)}} \frac{\partial D}{\partial x_{\sigma'(2)}} \frac{\partial E}{\partial x_{\sigma'(3)}} \right. \\
&\quad \left. + \frac{\partial C}{\partial x_{\sigma'(1)}} \frac{\partial^2 D}{\partial x_{\sigma'(2)} \partial x_{\sigma(3)}} \frac{\partial E}{\partial x_{\sigma'(3)}} + \frac{\partial C}{\partial x_{\sigma'(1)}} \frac{\partial D}{\partial x_{\sigma'(2)}} \frac{\partial^2 E}{\partial x_{\sigma'(3)} \partial x_{\sigma(3)}} \right), \tag{18}
\end{aligned}$$

$$\begin{aligned}
\{\{A, B, C\}, D, E\} &= \sum_{\sigma, \sigma'} (-1)^{\varepsilon(\sigma) + \varepsilon(\sigma')} \left(\frac{\partial^2 A}{\partial x_{\sigma(1)} \partial x_{\sigma'(1)}} \frac{\partial B}{\partial x_{\sigma(2)}} \frac{\partial C}{\partial x_{\sigma(3)}} + \frac{\partial A}{\partial x_{\sigma(1)}} \frac{\partial^2 B}{\partial x_{\sigma(2)} \partial x_{\sigma'(1)}} \right. \\
&\quad \left. \frac{\partial C}{\partial x_{\sigma(3)}} + \frac{\partial A}{\partial x_{\sigma(1)}} \frac{\partial B}{\partial x_{\sigma(2)}} \frac{\partial^2 C}{\partial x_{\sigma(3)} \partial x_{\sigma'(1)}} \right) \frac{\partial D}{\partial x_{\sigma'(2)}} \frac{\partial E}{\partial x_{\sigma'(3)}}, \tag{19}
\end{aligned}$$

$$\begin{aligned}
\{C, \{A, B, D\}, E\} &= \sum_{\sigma, \sigma'} (-1)^{\varepsilon(\sigma) + \varepsilon(\sigma')} \frac{\partial C}{\partial x_{\sigma'(1)}} \left(\frac{\partial^2 A}{\partial x_{\sigma(1)} \partial x_{\sigma'(2)}} \frac{\partial B}{\partial x_{\sigma(2)}} \frac{\partial D}{\partial x_{\sigma(3)}} + \frac{\partial A}{\partial x_{\sigma(1)}} \right. \\
&\quad \left. \frac{\partial^2 B}{\partial x_{\sigma(2)} \partial x_{\sigma'(2)}} \frac{\partial D}{\partial x_{\sigma(3)}} + \frac{\partial A}{\partial x_{\sigma(1)}} \frac{\partial B}{\partial x_{\sigma(2)}} \frac{\partial^2 D}{\partial x_{\sigma(3)} \partial x_{\sigma'(2)}} \right) \frac{\partial E}{\partial x_{\sigma'(3)}}, \tag{20}
\end{aligned}$$

$$\begin{aligned}
\{C, D, \{A, B, E\}\} &= \sum_{\sigma, \sigma'} (-1)^{\varepsilon(\sigma) + \varepsilon(\sigma')} \frac{\partial C}{\partial x_{\sigma'(1)}} \frac{\partial D}{\partial x_{\sigma'(2)}} \left(\frac{\partial^2 A}{\partial x_{\sigma(1)} \partial x_{\sigma'(3)}} \frac{\partial B}{\partial x_{\sigma(2)}} \frac{\partial E}{\partial x_{\sigma(3)}} \right. \\
&\quad \left. + \frac{\partial A}{\partial x_{\sigma(1)}} \frac{\partial^2 B}{\partial x_{\sigma(2)} \partial x_{\sigma'(3)}} \frac{\partial E}{\partial x_{\sigma(3)}} + \frac{\partial A}{\partial x_{\sigma(1)}} \frac{\partial B}{\partial x_{\sigma(2)}} \frac{\partial^2 E}{\partial x_{\sigma(3)} \partial x_{\sigma'(3)}} \right). \tag{21}
\end{aligned}$$

Substituting equations (18)-(21) into (4), we find that the super Nambu-Poisson bracket (13) satisfies the generalized FI condition (7) with the substitution $[\ , \] \rightarrow \{ \ , \ \}$.

Let us take

$$\begin{aligned}
L_m^i &= \sqrt{z} \exp[(i - \frac{1}{2})x - 2my], \\
\bar{L}_m^i &= \theta_1 \theta_2 \sqrt{z} \exp[(i - \frac{1}{2})x - 2my], \\
h_r^{\alpha + \frac{1}{2}} &= \theta_1 \sqrt{z} \exp[(\alpha - \frac{1}{2})x - 2ry], \\
\bar{h}_r^{\alpha + \frac{1}{2}} &= -\theta_2 \sqrt{z} \exp[(\alpha - \frac{1}{2})x - 2ry].
\end{aligned} \tag{22}$$

It is noticed from (22) that L_m^i is the generator of w_∞ 3-algebra presented in [17]. Substituting the generators (22) into (13), we obtain the classical super w_∞ 3-algebra (12).

4 Concluding Remarks

We have investigated the SHOV 3-algebra. By applying the appropriate scaling limits on the generators, we obtained the super w_∞ 3-algebra. We also found that this super w_∞ 3-algebra satisfies the generalized FI condition. To present the realization of the super w_∞ 3-algebra, we defined a super Nambu-Poisson bracket which satisfies the generalized skewsymmetry, Leibniz rule and fundamental identity, but the derivative term with respect to the Grassmann variables are not involved in the 3-bracket. Moreover, we tried to introduce the Grassmann derivative term in the super Nambu-Poisson bracket such that the super w_∞ 3-algebra can be realized. Unfortunately, we did not succeed in finding any one. Whether there exists this kind of super Nambu-Poisson bracket still deserves further study. Furthermore the application of this super w_∞ 3-algebra in physics should be of interest.

Acknowledgements

This work is partially supported by NSF projects (10975102, 10871135 and 11031005).

References

- [1] Y. Nambu, Generalized Hamiltonian dynamics, *Phys. Rev. D* **7** (1973) 2405.
- [2] J. Bagger and N. Lambert, Modeling multiple M2's, *Phys. Rev. D* **75** (2007) 045020 [hep-th/0611108].
- [3] J. Bagger and N. Lambert, Gauge symmetry and supersymmetry of multiple M2-branes, *Phys. Rev. D* **77** (2008) 065008 [arXiv:0711.0955].
- [4] S.A. Cherkis and C. Sämann, Multiple M2-branes and Generalized 3-Lie algebras, *Phys. Rev. D* **78** (2008) 066019 [arXiv:0807.0808].
- [5] G. Papadopoulos, M2-branes, 3-Lie algebras and Plücker relations, *JHEP* **05** (2008) 054 [arXiv:0804.2662].
- [6] A. Gustavsson, Algebraic structures on parallel M2-branes, *Nucl. Phys. B* **811** (2009) 66 [arXiv:0709.1260].
- [7] P.M. Ho and Y. Matsuo, M5 from M2, *JHEP* **06** (2008) 105 [arXiv:0804.3629].
- [8] P.M. Ho, Y. Imamura, Y. Matsuo and S. Shiba, M5-brane in three-form flux and multiple M2-branes, *JHEP* **08** (2008) 014 [arXiv:0805.2898].
- [9] C.H. Chen, K. Furuuchi, P.M. Ho and T. Takimi, More on the Nambu-Poisson M5-brane theory: scaling limit, background independence and an all order solution to the Seiberg-Witten map, *JHEP* **10** (2010) 100 [arXiv:1006.5291].
- [10] L. Takhtajan, On foundation of the generalized Nambu mechanics, *Commun. Math. Phys.* **160** (1994) 295 [hep-th/9301111].
- [11] P. Gautheron, Simple facts concerning Nambu algebras, *Commun. Math. Phys.* **195** (1998) 417.

- [12] J.A. de Azcárraga and J. M. Izquierdo, Topics on n-ary algebras, J. Phys: Conference Series **284** (2011) 012019 [arXiv:1102.4194].
- [13] T.L. Curtright, D.B. Fairlie and C.K. Zachos, Ternary Virasoro-Witt algebra, Phys. Lett. B **666** (2008) 386 [arXiv:0806.3515].
- [14] T.A. Larsson, Virasoro 3-algebra from scalar densities, arXiv:0806.4039.
- [15] T. Curtright, D. Fairlie, X. Jin, L. Mezincescu and C. Zachos, Classical and quantal ternary algebras, Phys. Lett. B **675** (2009) 387 [arXiv:0903.4889].
- [16] C.N. Pope, L.J. Romans and X. Shen, The complete structure of W_∞ , Phys. Lett. B **236** (1990) 173.
- [17] S. Chakraborty, A. Kumar and S. Jain, w_∞ 3-algebra, JHEP **09** (2008) 091 [arXiv:0807.0284].
- [18] J. Palmkvist, Three-algebras, triple systems and 3-graded Lie superalgebras, J. Phys. A: Math. Theor. **43** (2010) 015205 [arXiv:0905.2468].
- [19] N. Cantarini and V.G. Kac, Classification of simple linearly compact n-Lie superalgebras, arXiv:0909.3284.
- [20] Y.L. Daletskii and L. Takhtajan, Leibniz and Lie algebra structures for Nambu algebra, Lett. Math. Phys. **39** (1997) 127.
- [21] M. Sakakibara, Notes on the super Nambu bracket, Prog. Theor. Phys. **109** (2003) 305, [math-ph/0208040].
- [22] D.V. Soroka and V.A. Soroka, Odd Nambu bracket on Grassmann algebra, hep-th/0608052.
- [23] D.V. Soroka and V.A. Soroka, Nambu-like odd brackets on supermanifolds, arXiv:0811.3074.
- [24] C.Z. Zha and W.Z. Zhao, The q deformation of super high-order Virasoro algebra, J. Math. Phys. **36** (1995) 967.
- [25] C.N. Pope and X. Shen, Higher-spin theories, w_∞ algebras and their super-extensions, Phys. Lett. B **236** (1990) 21.